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THESIS

~~RESEARCH STUDIES IN~~ THE TEACHING OF SECONDARY MATHEMATICS

SUBMITTED BY

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(S.B. IN ED., BOSTON UNIVERSITY)

IN PARTIAL FULFILMENT OF REQUIREMENTS FOR

THE DEGREE OF MASTER OF ARTS

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Introduction

The purpose of this study is to make a survey of the recent literature in the field of secondary mathematics in order to determine to what extent present practice is based on research and experiment.

The outlook is hopeful. The number of research studies is large, and the literature written by experts in the field is extensive. Teachers, specialists in mathematics, and college professors who train teachers seem to share the opinion that the experimental studies so far have touched only the surface; that valid aims must first be set before the content used to obtain these aims can be selected; that the testing program is becoming more adequate, and that approaches are being made to the problem of fitting the mathematics program to the needs of the pupils.

The plan of the thesis is simple. In Chapter I, pages 3-46, the fourteen doctors' dissertations and masters' theses on secondary mathematics appearing between 1926 and 1931 are reviewed and evaluated. Chapter II, pages 47-98, reviews and evaluates the articles in the six yearbooks of the National Council of Teachers of Mathematics, 1926-1931 inclusive. Chapter III, pages 99-116 reviews articles appearing in School and Society, School Review, Teachers' College Record, and Journal of Educational Research, 1926-1931 inclusive. The summary, pages 117-118, gives some of the outstanding conclusions which may be obtained from this study.

Chapter I

Reviews of Doctors' Dissertations and Masters' Theses.

Curriculum

1.

The Teaching of General Mathematics in the Secondary Schools of the United States.
Clarence McCormick, Ph.D. Columbia University, 1929
Teachers' College Contribution No. 386
173 pages.

This dissertation presents a study of the development and present status of general mathematics. It is the result of a wide study of (1) courses of study of cities and states, (2) of text books for the seventh, eighth, ninth, and tenth grades, (3) of a questionaire sent to teachers, (4) of a study of publications of specialists in mathematics.

A history of the general mathematics movement is given with reviews of seven studies on teaching general mathematics. The evidence from all of these seems to be significant and to warrant the following conclusions:

1. There is not much difference in algebraic ability of pupils who have finished one year of algebra or one year of general mathematics.
2. The pupils with two years of general mathematics in the high school excelled those with one year of algebra and one year of plane geometry in these experiments.

3. In the only experiment comparing two groups of third year high school pupils, the general mathematics group ranked considerably higher in both algebra and geometry.
4. The college students who have had two years of of general mathematics in the high school seem to make grades in mathematics at least as high and probably higher than the students who have had a year of algebra and a year of plane geometry.

A study of thirty-two seventh and eighth grade text books shows that the trend is toward general mathematics in the junior high school.

A study of 46 text books for the ninth grade shows that authors tend to include more material from the different branches of mathematics but that algebra continues to be the chief subject. There is some tendency to include a unit of demonstrative geometry. A unit of numerical trigonometry is always included.

A study of 28 tenth grade books shows that there are no significant changes in content being made at the present time. The subject is practically static.

To determine the extent of general mathematics an investigation was made in July, 1928 of all the printed courses of study available at the Bureau of Curriculum, Teachers' College, Columbia University. In January,1929 the writer sent to the same cities and

states for the new courses of study. Courses of study of 13 cities and 9 states , and that recommended by the College Entrance Examination Board were obtained. The tables of results show:

1. The prevailing type of mathematics in the ninth grade of the junior high school courses is general mathematics.
2. In the four year high school the work of the ninth grade is more traditional.
3. The methematics courses of study for the tenth grade are much more traditional than for the ninth grade in either type of school organization.

To get a view of the attitude teachers hold toward general mathematics, questionaires were sent to teachers of mathematics asking which type (general or traditional) of mathematics was taught, and which was preferred and why. There were 216 replies from 67 different schools. Most of the replies show that the influence of the general mathematics is breaking up the strict separation of mathematics into different branches and is establishing greater unity.

To obtain the opinion of specialists in the teaching of mathematics the writer investigated 86 books by 62 different authors.

The reply of William Betz, specialist in mathematics, Rochester, New York is quoted:

> "A perfect solution of the question of general mathematics is, of course, impossible within the customary time limits. The crux of the whole problem is not merely the new content, but especially the time schedule. It is absurd to talk merely about the ninth and tenth years. General mathematics should begin in the kindergarten and extend through the university. At present, it has no chance whatever, so long as we insist upon the absurd notion that it is a question merely of the upper high school."

The opinion of specialists in the teaching of mathematics greatly favors a more general mathematics course.

General Conclusions.

I. There is no definite agreement among mathematicians and educators as to what constitutes general mathematics, and there is no text book which meets general approval.

II. The traditional mathematics of the 7th, 8th, and 9th years is gradually being replaced by a more general type. In the Junior High School the mathematical work for the ninth year is of a more general type, while in the four year high schools the mathematics courses are more traditional.

III. General mathematics is as good preparation for college as the traditional mathematics.

IV. General mathematics is favored by many teachers because of the wide variety of information given, the interest created, and because of its practical value.

V. Traditional mathematics is favored by many because of the more thorough knowledge given, its better organization, and the belief that it gives a better preparation for college.

This is a valuable Library Study. The author draws his conclusions from a study of text books, previous experiments, and opinions of specialists in mathematics and not from actual experiments under controlled conditions. I believe that controlled experiments should determine the best material to be included in a general mathematics course, since opinion now seems to favor mathematics of a more general type.

A Study of Achievement in General Mathematics.
By June May Crow. A.M. 1927.
University of Washington, Seattle, Washington.
75 pages.

The purpose of this study is to answer the question, "Do the pupils who have as a foundation in mathematics the new organization of subject matter possess in the end a better knowledge of mathematics than those who had as a foundation the old organization of subject matter?"

The new organization would introduce the easier parts of algebra and geometry earlier and spread arithmetic, algebra, and geometry over three years work. The regular organization consists of arithmetic in the seventh and eighth grades, algebra in the ninth grade, and geometry in the tenth grade as entirely separate units.

In order to compare achievement in mathematics between pupils who had studied general mathematics and those who had studied algebra, standardized tests were given, and the results obtained were compared. The tests used were the same for all; they were given to pupils in the first half of their junior year, pupils who had studied either general mathematics through grade IX, or who had completed algebra and geometry, and who had not continued their mathematics instruction. All pupils had completed geometry.

The Hotz First Year Algebra Scale and the Minnick Geometry Tests were chosen. There were two series in each scale. A total number of 562 tests were given during the middle of the year, 1926-'27. Most of the students who had taken plane geometry were taking advanced work in mathematics, and so were excluded from these tests.

All of the papers were sorted into two groups, Group I containing papers of pupils who had studied general mathematics in grade IX, and Group II containing papers of pupils who had studied algebra in Grade IX.

The standard scores of the different tests are included in the tables. The formulas used for the Standard Deviation of Difference and for the Experimental Coefficient are from *McCall's, How to Measure in Education. Results are also interpreted in terms of chances which have been computed from the experimental coefficient by McCall. There are nine tables and three graphs, which show the results of the tests given.

Conclusions.

This study shows that general mathematics gives as much knowledge as the regular organization of mathematics, but the results of the study do not show that general mathematics is superior in achievement to the mathematics of the traditional type.

If we conclude that the new type of mathematics is as good as the old, then the emphasis in geometry can change from demonstration of theorems to the wide field of problems offering opportunity for interesting and practical use of theorems studied.

Since the pupils who have studied general mathematics show equal ability with the algebra pupils, and since

*McCall, William A. How to Measure in Education. Macmillan Co. N.Y., 1922.

the new type of subject matter offers a splendid opportunity for motivation, introducing interesting material for the child and broadening his view of the subject, we conclude that general mathematics has a permanent place in the curriculum.

Comment.

This is a minor research. The standardized tests used may favor the group in traditional mathematics since they are for the most part based on this traditional type. If this is true, general mathematics may be decidedly better than the traditional type of mathematics.

The greater number of pupils continued with mathematics in the junior year, and so were excluded from these tests. These excluded pupils seem to form an important group and should doubtless be included in the tests if conclusions are to be considered valid.

There were hardly enough tests given to place the results as much better than chance opinion.

Methods
3.

Motivating the Study of Geometry.
Louise Barbour Eddy. A.M. University of Chicago. 1930
149 pages.

This study traces (1) the historical development of ways of motivating the study of plane geometry, and (2) the motivating devices used in text books, and (3) changes in tendencies of the last forty-five years.

Material was collected from the following sources:

1. 25 text books on education and psychology
2. reports of important committees
3. yearbooks of National Association of the teachers of mathematics
4. yearbooks of the National Education Association
5. mathematics magazines and journals
6. 45 text books of plane geometry.

There are 33 tables in this article.

Conclusions.

Teachers are making an effort to interest the pupils in geometry. The applied problem was used for motivation as early as 1850. Increase of efforts toward more motivation have followed the establishment of important committees. The years 1895,1910, and 1924 are decided lines of demarcation.

Research studies have shown that interest is an aid in learning.

Writers on education suggest many devices for motivation. The ones most frequently mentioned are the

challenge, aims, penalties,collections, and practical activities.

Text books offer most frequently applied problems, historical material , usefulness of subject, arrangement of subject matter.

The trend is toward a greater variety of devices, use of new type tests, challenges, and optional material.

New inventions offer a field for interesting applications.

Comment.

This is a valuable library study of extensive material, which justifies the conclusions drawn.

No attempt at evaluation and comparison of motivating material is made in the study. The study does not touch on suggestions for a motivated unit of instruction. The material suggested consists only of short exercises. It does show, however, that teachers realize the importance of interest in learning, and the importance of emphasizing geometry as an appreciation unit.

4

An Experiment to Determine the Relative Merits of a General Rule versus Special Rules in Teaching Algebraic Special Products.
Harry Harding Tothero. A.M. Pennsylvania State College. 53 pages.

The purpose of this study is to get facts in regard to the relative achievement of two groups, the first taught using the traditional five rules, the second taught using the general rule which really covers all five traditional rules. A subordinate experiment determines the relative abilities of students to factor the trinomials which are the products of binomials. A third, but subordinate purpose is to observe results obtained by teaching simultaneous equations to one section by substitution only, and to the second section by both substitution and addition and subtraction methods.

This experiment was carried on during a two year period. During 1928-'29 there were three sections. During 1929-'30 there were also three sections, one taught by the control method (traditional) and two taught by the experimental method. In matching the groups intelligence quotients and school grades were used, and computed. An effort was made to have the sections as nearly matched as possible.

In the control group the experiment was begun with a lesson on Case I "The product of two binomials having a common term." The lesson was developed inductively.

Then special products were taught, also inductively.

The "Experimental Group" began with the general case, "The product of two binomials whose corresponding terms are similar."

In factoring the work in the "Experimental Section" began with the general case and all other cases except the monomial factor were taught by the same rule.

In teaching simultaneous equations one group used only the method of solving by substitution, the other used both methods, substitution and addition and subtraction. Coefficients of correlation of composite scores with each of the tests were calculated.

There are 28 tables and three graphs in this article.

The study was suggested by Thorndike's statement that one axiom of education is, "Do not form two or more habits where one will do as well."

Conclusions.

In the three tests on special products given during the first year all correlations of composite scores are considerably higher than the correlations for the control group. The average difference is +.254.

In the three tests on special products during the second year of the experiment all correlations for the experimental group were higher except one where the difference is .07 in favor of the control group. The

average difference is +.093.

Of these six tests five show conclusively that the general rule is better than the special rules. In the factoring test the correlation of the control group exceeds that of the experimental group by .321. In the test on simultaneous equations the results favor the group that was taught both the substitution and addition methods. The difference is .659 (in correlation)

Comment.

The study does not state how many pupils were in each section. There were six sections, three each year. The experiment seems well organized, but not extensive enough to warrant final conclusions.

The results of this study, however, involving the six sections show that the general rule is better than special rules in teaching algebraic special products.

Correlation

5

Training in High School Mathematics Essential for Success in Certain College Subjects.
Allan Ray Congdon. Ph.D. Columbia University. 1930.
Teachers' College Contribution, No. 403.
102 pages.

This is a valuable library study, the purpose of which is to determine the extent of mathematics necessary for success in certain college subjects that do not presuppose college mathematics.

A study of Stuart's text, Physics, A Text Book for Colleges, is the basis of data in the following analysis:

I. A careful word count for mathematical vocabulary.

II. The symbolism used in the text was examined and tabulated and compared with that of four other texts in physics and five in chemistry and 15 in statistics.

III. By means of an intensive study of the entire text, including complete solution of the 572 problems, the facts, concepts, skills, general processes, and methods of procedure were analysed and tabulated.

IV. The Congdon Algebra Test designed for pupils who had completed one and a half years of high school algebra was given to 1000 pupils, the purpose being to determine objectively the degree of difficulty of various formulas.

This study contains 15 tables. There is very careful and complete explanation of all procedure.

Findings.

1. The mathematical vocabulary used in college physics is extensive. It seems advisable to place more emphasis on the following words: axis, rotation, clockwise and counter clockwise motion, positive and negative, constants and variables, maximum and minimum, rates and speeds, and words connected with variation and proportionality.
2. The symbolic language used in college subjects is quite different from that used in high school mathematics.
3. The arithmetic computations found in college text books are more complicated than those now advised for courses in arithmetic. Very small numbers and very large numbers, decimal fractions, and common fractions are widely used.
4. There are comparatively few geometric facts used.
5. The ability to make accurate scale drawings is essential.
6. The ability to represent relationships by graphs and to read off values from graphs is essential.
7. The course in numerical trigonometry as now taught in high school seems sufficient.
8. The algebraic equations are not so difficult , but are unusual in that they are not the customary

or habitual form of algebra text books. The answers are usually fractional, varying from one to seven significant digits at the right of the decimal point.

9. There are no complicated polynomials, parentheses, complicated fractional equations, or complicated radicals in the texts examined.

10. 23 general processes and methods of procedure are listed in the order of frequency of use. Two are always used, (1) critical reading of problem to get exact meaning, (2) roughly checking answer. The third occurs 428 times, (3) determining appropriate formula and making numerical substitution in it.

11. It seems that the following topics in regard to subject matter should be given more emphasis:

    A. Accuracy in computation with especial reference to decimal points and signs.
    B. Manipulation of formulas using capital letters, Greek letters, primes and subscripts.
    C. Simple fractional equations.
    D. Simple radical equations.
    E. Simple applications of parentheses.
    F. Simultaneous equations in fractional forms.
    G. Fundamental operations with standard form numbers.
    H. Solving for a ratio between two numbers.

I. Proportion and variation as two phases of the same subject.

J. Scale drawings.

12. The general processes and methods of procedure listed according to frequency of use in Stewards text

A. The critical reading of the problem in order to get the exact meaning. This is always used.

B. Roughly checking the answer to see whether it is reasonable, always used.

C. Determining the appropriate formula and and making the numerical substitution in it. (428 times)

D. Changing from one unit of measure to another by means of a known ratio. (239 times)

E. Reduction of data to the proper units of measure before substitution in the appropriate formula. (154 times)

F. Numerical substitution in the appropriate verbal law or definition. (104 times)

G. Translation of a verbal law or definition into a mathematical formula.(85 times)

H. Transferring from the type or illustrative problem as stated, and then planning an analogous solution. (64 times)

I. Unitary analysis. (64 times)

J. Mathematical statement of a law which has not been espressed in words.(63 times)

K. The resolution of a problem into two or more component parts, each part being dependent upon the one preceding. (55 times)

L. A certain amount of generalization from the type or illustrative problem in order to transfer to the new problem, and then the planning of the analogous solution. (53 times)

M. Making use of a fact, technique or procedure that has been suggested by a preceding problem. (27 times)

N. The use of the index of the text to find laws and definitions previously stated. (27 times)

O. Practical interpretation of a law or relationship symbolically stated. (7 times)

P. Solution of a problem for the ratio between two quantities when the value of neither quantity is known. (19 times)

Q. Discarding irrelevant data. (14 times)

R. Interpretation of a formula when certain values or limitations are assigned to a letter or letters of the formula.(14 times)

S. Interpretation of a negative root of an equation that has been established to solve a practical problem .(13 times)

T. Using a verbal rule as a stimulus to a process carried on in imagination. (7 times)

U. The resolution of a problem into two or more

independent parts which are finally combined for the required answer. (6 times)

V. Interpretation of the meaning of a negative value of a letter which is substituted in a formula. (5 times)

W. Generalization from a numerical problem to a literal formula. (3 times)

Comment

The author infers that in teaching algebra we should have as one aim, preparation for work in college. Research studies gave shown that it is better to teach in high school what pupils can understand at the time, and what they need at the time. This study, however, suggests some interesting work which can be included easily in the high school course, and which would enrich a pupil's appreciation of the value of mathematics.

A fact not to be overlooked is that the mathematics used in college physics is simple and not complicated. This may be an argument for placing less time on drill on complicated exercises, thus allowing more time for interesting applications of mathematics in other fields.

Relation of Success in Mathematics to Success in Physics and Chemistry in High School.
James H. Winegardner. A.M. Leland Stanford Junior University. 1929
60 pages.

The purpose of this study is to determine the relation of success in algebra and geometry to success in physics and chemistry in high school, so the results may be of use as a motivation for high school mathematics.

Three types of evidence were used.

1. Grades of 450 pupils of Oakland Technical High School who had taken algebra and geometry and and either physics or chemistry or both.
2. Reviews of 12 previous investigations;
3. Collection of 44 expert opinions (people of long experience and outstanding in the field of teaching physics and chemistry.)

Final semester grades of the 450 students were recorded in the following subjects: first year algebra, plane geometry, U.S. history, physics, chemistry, and advanced algebra. The I.Q. was also noted. Coefficients of correlation between final grades in these subjects were obtained. Intelligence quotients were also correlated with physics and with chemistry.

There were seven tables, one percentage chart and two scattergrams.

Conclusions.

Physics. In the present study there is low correlation betw een physics and other subjects. The algebra-geometry

combination correlated with physics gives the highest ratio (r=0.303). Low correlation is due to the fact that there is was a very high percentage of failures in physics.

The twelve previous investigations show a decided positive relation between high quality work in mathematics and high grade work in physics.

The evidence as summed up from the opinions of 44 experts is that mathematics is necessary to an understanding of both physics and chemistry.

Chemistry.

The algebra-geometry combination correlated with chemistry gives a ratio of 0.579. The same combination correlation with history gives an ratio of .499. Difference of .080 in favor of chemistry. The algebra-geometry combination is the best criterion of success in chemistry.

Previous studies have little to offer as evidence concerning relation of success in mathematics to success in chemistry.

Comment.

This study does not show that success in mathematics assures success in physics. Since algebra and geometry are generally presupposed for entrance to the physics course, it would be difficult to study the success of physics pupils who had not studied high school mathematics.

Content

7

A Study of Problem Material in High School Algebra.
Jesse Jerome Powell. Ph.D.Columbia University. 1930
Teachers' College Contribution No. 405.
39 pages,together with 42 pages of selected problems.

The purpose of this article is to select a list of 500 of the best problems in elementary algebra available at present, the selection being based on(a) genuineness, (b) importance, (c) interest .

Three thousand original problems were submitted by experts. Thirteen hundred were selected and divided into groups of 150 each, each group containing a number of every type. These were sent out to 500 trained and experienced teachers of algebra, who were instructed to rate each problem on the scale of 0 to 10 on the basis of (a) genuiness, (b) importance, (c) interest.

200 problems were selected from those rated five or above in genuiness and importance. In order to study interest- determining factors some problems of all types were added whether they ranked high in the two factors named above or not.

These problems were sent out to pupils just completing ninth grade algebra. Pupils were told to set up equation and indicate whether they liked the problem "very much", "fairly well", or "not at all".

Findings.

1. Problems considered genuine were graded high in estimated interest by teachers.

2. Problems considered important by teachers ranked high in interest estimated by teacher.
3. There was a definite tendency toward low correlation between importance of problems and interest expressed by pupils.
4. In associating pupil interest and interest estimated by the teacher the correlation coefficient is 0.11. There is practically no association between interest as expressed by pupil and as estimated by teacher.
5. Pupils do not lose interest when new and difficult problems are introduced, provided there are not too many of these.

Appendices.

Appendix A is devoted to methods of teaching algebra.

Appendix B gives the following lists of problems:

A. 102 problems selected with reference to interest as registered by pupils.
B. 123 problems selected with reference to their importance.
C. 241 problems, a miscellaneous group in one and two unknowns.

Conclusions.

In the opinion of expert teachers the chief elements of interest of a problem lie in its application to life. They consider problems dealing with economic

situations most important, while age, angle, digit, clock, and puzzle problems have about zero evaluation.

The greatest emphasis is now being placed on the most common experiences of the average citizen. Problem material should be selected from that which is used and ought to be used by the average citizen. Teachers should set up a situation which seems real to the pupil. Pupils are more interested in problems that express activities of youth and that offer a challenge, and that introduce the puzzle element. They do not prefer problems involving adult economic activities. Reasonableness is probably a minor element.

Comment.

This is a valuable minor research. The interest factor is difficult to determine. Pupils are inclined to grade high on a questionaire those problems which they can easily do. The author has compiled a list of interesting and useful problems.

Ability of High School Pupils to Select Essential Data in Solving Problems.
Benjamin W. Daily. Ph.D. Columbia University. 1925.
Teachers' College Contribution. No. 190.
85 pages.

The purpose of this study is to determine whether pupils of first year algebra are able to select essential data and use it and to realize lack of sufficient data. Problems in life situations are complicated.

Fourteen text books were scanned and verbal problems studied. Questionaires were sent to text book writers and to teachers of high school algebra.

Tests were given to 450 pupils from twelve schools. The tests contained problems familiar to pupils and also problems containing superfluous or insufficient data.

An experiment was made on a group of 45 pupils to determine effects of appropriate instruction.

An experiment was made with three equivalent groups to measure the effect of four weeks special instruction. They were tested by algebra and non-mathematical tests. Forms Alpha and Beta contained both regular and special problems. Mathematical, non-mathematical and control groups were tested. The control group had no special instruction.

This study contains 20 tables and 15 scattergrams. Findings.

Text book writers do not use problems containing superfluous or insufficient data. Teachers feel that problems of this type confuse children.

Tests show that pupils are in the habit of having all necessary data given and no more than is necessary. Pupils do not question the adequacy or relevance of the data given. They are baffled by unusual situations. Problem work is not good even on problems familiar to the pupils. A small amount of training, however, results in decided improvement in selecting data. The trained group also showed decided improvement in non-mathematical tests.

Comment.

The experiment involves too few pupils and extends over too short a time to be very valuable. Class room teachers find that pupils are often unable to understand and use the data correctly which is essential. Therefore, why give pupils more? This study shows that problem work is not good even on problems familiar to the pupils. Making the problems more complicated is probably not the way to improve the situation.

Objectives

9

An Examination and Critical Evaluation of the Elements of Geometry Teaching in the Secondary Schools.
Wallace Theodore Wait. Ph.D. University of Washington, Seattle, Washington.
200 pages.

The purpose of this study is to examine and criticize by the study of widely used text books and of the reports of important mathematical committees the following topics:

1. Objectives of geometry teaching.
2. Methods of teaching whereby aims can be realized.
3. Methods of evaluating results of geometry by use of diagnostic and achievement tests.
4. Comparison and evaluation of the most widely used geometry text books.
5. Investigation of the mathematical preparation of geometry teachers.

125 replies to questionaires sent to teachers of geometry in Washington, Idaho, South Dakota, and California were obtained. This questionaire had four parts.

1. Statements of 25 aims, which teachers were asked to grade in regard to importance.
2. 25 statements of class procedure to be rated as Part I.
3. This part was designed to determine the texts used and to obtain criticism of the strong and weak points of each. It was also to find what supplementary material was in use and what

standardized tests were being used.

4. This part asked for data concerning experience, education, and mathematical scholarship of teachers.

The values of the study of geometry are based on the following sources of opinion:

1. Opinions of mathematicians from early time to the present.
2. Report of the International Committee on Mathematics Teaching made in 1912.
3. Report of Committee on the Reorganization of Mathematics made in 1923.
4. Returns from a questionaire sent to professors of mathematics in colleges and universities who are conducting classes in the teaching of mathematics in secondary schools.
5. Returns from questionaires sent to geometry teachers.

Conclusions.

I. Values.

A. There is a decided difference of opinion as to the importance of the 25 statements of aims.

B. High school teachers place much emphasis on the completion of the material in course of study within the time allowed. This shows that as a group they follow the text book very closely. College professors place this objective very low.

C.

C. Professors in colleges still consider disciplinary values of major importance, while high school teachers seem to place little emphasis on them.

D. College mathematics professors rate the cultural and ethical aims low, and do not place much emphasis on the historical and avocational objectives.

E. High school teachers rate very high the preparation for college entrance examinations, while professors of mathematics place much less stress on this point.

G. The two groups seem to have different points of view, each group centered in its own particular problems.

II. Methods of geometry instruction.

A. There is a wide difference of opinion between the high school teachers and the college professors over the method of classroom procedure and curriculum arrangement.

B. The college group agrees more closely than the high school group with the recommendation of the National Committee on Mathematics Requirements.

C. Teachers' courses in mathematics should be investigated to determine

1. what methods prospective teachers should be taught to emphasize

2. Means by which these desirable methods can be put into effect in the teacher's experience.

III. Geometry textbooks in secondary schools.

A. The results of the questionaire show that Durrell and Arnold was used by 45% of the teachers who replied.

B. Most teachers indicate very little use of supplementary material.

C. Teachers show a tendency to be dissatisfied with whatever textbook they are using.

IV. Teacher Preparation.

A. The results of the questionaire sent to college professors of mathematics and to teachers of geometry show the general level of mathematical preparation to be about the same as the mean level of the standard set by the college professors of mathematics.

B. From the standpoint of teacher training geometry seems to occupy a better position than the physical sciences.

C. Failure to produce expected results must point to other sources of failure than teacher preparation.

D. There appears to be no recognition of a need for differentiation between preparation for a scholar of geometry and that for a teacher of geometry.

V. Geometry testing.

A. The standardized geometry tests are evidently little used.

B. The most general criticism of the objective type tests is that they do not test ability to demonstrate theorems or to reason logically.

C. Tests of more than a very limited use are not available.

Comment.

There were only 125 replies to the questionaire sent to high school teachers. The number of returns from college professors of education is not given. Therefore adequate conclusions can not be drawn, since the results are mere opinions.

The Fundamental Skills of Algebra.
John Phelps Everett. Ph.D. Columbia University.
Teachers' College Contribution. No. 324.
102 pages.

The purpose of this thesis is to give the conclusions gained from analyzing written tests of several hundred pupils. The results are not tabulated, but applications to classroom procedure are given.

111 pupils took tests consisting of 285 examples at 18 different occasions. The ability range was from I.Q. 96 to I.Q. 158.

549 pupils from 35 different schools took a test of 94 selected algebraic exercises just before taking College Entrance Board examinations.

There are no tables given and no results of tests are given.

Findings.

In analyzing the skills necessary to solve a problem on which only 27.2% of the pupils succeeded, the same pupils did succeed in the skills of addition, subtraction, multiplication, and division. Abilities in the application and organization of the manipulative skills seem to be the most difficult to attain.

While in arithmetic there are 1680 number combinations spread over a period of six years, in algebra the number combinations are infinite.

In addition to the manipulative skills of addition, subtraction, multiplication, and division, the author lists 44 skills of understanding which can be taught.

Too much time is spent on manipulative skills, and not enough on the skills of understanding.

Comment.

This is a library study. Educators seem to agree that more time should be spent on the skills of understanding. This is ,however, the author's opinion, and since no data are given we are unable to form conclusions from the study.

Text Book Analyses

11

Comparison of Certain Plane Geometry Texts of the Two Periods, 1899-1904 and 1925-1929.
Lewis D.Prag. A.M. New York University. 1930
50 pages, 11 tables.

The purpose of this study is to determine how text books now in use differ from those of the period 1899-1904.

Sources.

Four text books of the former period were selected, also four text books now in use, and comparisons made on the following phases:

A. Descriptive phases
   1. Preface
   2. Approaches to propositions.
   3. Comparison of propositions.
   4. Use of trigonometry

B. Numerical phases.
   1. Introduction
   2. Number of propositions
   3. Propositions after which problems are omitted.
   4. Number of problems.
      (a) Total (b) applied problems (c) laboratory problems.
   5. Number of figures.

The texts chosen were -

Old:
1. Wentworth, G.A. 1899 Plane and solid geometry
2. Milne, W.J. 1899 Plane and solid geometry
3. Schultze,A. and Sevenoak, F.L. 1901 Plane Geometry
4. Durrell,F. 1904 Plane and solid geometry

New: 1; Schultze and Sevenoak. 1925. Plane and solid geometry.
2. Clark,J.R. and Otis,A.S 1927. Modern Plane Geometry.
3. Hawkes, Luby and Tonton 1929. New Plane Geometry.
4. Nyberg, J.A. 1929 Plane Geometry

Data

Section II states abstracts from prefaces to express the authors' aims for geometry and to emphasize special features of the texts.

The approaches to propositions in the old texts were compared with the approaches in the new. The new texts give a more thorough approach to geometry as a whole and to certain propositions. The introductions in the old texts consist of a series of definitions with figures.

Typical propositions were compared. In the old texts a few propositions were incomplete while in the new texts many proofs are entirely omitted. There is a difference in the arrangement of proof. The propositions are the same in both. The arrangement of the proof is better in modern texts.

Trigonometry is not found in the old texts. It is found in the new texts, except Clark's.

The introductions to all texts were compared as to

number of pages)
number of problems) which precede Book I
number of figures)

The number of pages devoted to introductions in old texts range from 6 to 20; in the modern texts, 16-38.

There are four times as many problems in the new texts as in the old. The range in the old texts is from

o to 37; in the modern, 56 to 128. The range of the number of figures in introductions of old texts is from 3 to 27; in the modern texts from 39 to 67.

Modern texts initiate pupils gradually into geometry.

Number of propositions in old and modern texts.

Old Texts		Modern Texts	
Milne	173	Schultze(1925)	152
Wentworth	167	Clark	87
Schultze (1901)	163	Nyberg	94
Durrell	177	Hawkes	126
Average	170	Average	115

The average number of propositions in the modern texts is 68% of the average number in the old texts.

Table to show practice in application, given immediately following the propositions.

The number of propositions after which problems are omitted.

Old Texts		Modern Texts	
Milne	93	Schultze (1925)	27
Wentworth	118	Clark	12
Schultze(1901)	27	Nyberg	19
Durrell	91	Hawkes	25
Average	82	Average	21

Ratios of the number of propositions after which problems are omitted.

Old Texts		New Texts	
Wentworth	Ratio .71	Schultze (1925)	Ratio .18
Milne	.54	Clark	.14
Schul tze	.17	Hawkes	.20
Durrell	.51	Nyberg	.20
Average	.48	Average	.18

The modern texts have made much greater effort to introduce problems immediately following the proposition. The pupil uses facts just learned.

Problems were counted. The average of the total number of problems in the old texts is 63% of the average of the total number of problems in the modern texts. Since the number of propositions decreased, there is time for more problems.

There are practically no laboratory problems (i.e. paper cutting)in the old texts. The range is from 0 to 11. The range in modern texts is from 29 to 108.

Number of applied problems.

Old Texts		Modern Texts	
Milne	13	Schultze	153
Wentworth	14	Clark	107
Schultze	9	Nyberg	43
Durrell	40	Hawkes	202
Average	19	Average	126

The number in the old texts is 15% of the number in the new texts. The average number of figures in old texts is 36% of the average number of figures in modern texts.

Conclusions.

I. The prefaces of both old and modern texts emphasize the chief purpose of geometry as the development of mental power.

II. Several approaches to propositions in the modern text contain problems in addition to the usual definitions.

III. The propositions in the modern texts contain a short description of the method to be employed in the proof; the arrangement in the modern texts of proof is more outstanding.

IV IV. The modern texts contain an introduction to trigonometry.

V. In the modern texts there are increases of -

A. 150% in the number of pages of introduction

B. 300% in the number of problems in introduction

C. 167% in the number of figures in introduction

D. 60% in the number of problems

E. 148% in the ratio of the total number of problems to the number of propositions

F. 15 fold increase in the number of laboratory problems if Milne's text is not considered.

G. 663% in the number of applied problems.

H. 295% in the ratio of the number of applied problems to the total number of problems

I. 177% in the number of figures

J. 53% in the ratio of the number of figures to the total number of problems .

VI. There are decreases in the modern texts of -

A. 63% in the ratio of the number of propositions after which problems are omitted to the total number of propositions.

B. 32% in the number of propositions.

Comment.

This study of the textbooks of two periods is an answer to the argument that geometry texts have not changed greatly during the last 25 years. While the basic material is the same, the emphasis is now on interesting applications of facts learned.

12

Analysis of Elementary Algebra Textbooks for Comparison of Content and Emphasis of Material
Earl K.Stock. M.S. Pennsylvania State College. 1930
20 pages.

Ten text books of first year algebra are analyzed, the primary purpose being to determine and compare varying amounts of emphasis on the different topics of the subject. These text books were selected as being in common use and designed for a course in elementary algebra as generally conducted in the ninth year.

No summary of comparison of content and emphasis is given in this study.

The five tables give the number of drill problems, number of verbal problems and number of lines of explanation devoted to each of the following topics.

Alg. Sym.	Div. (gen.)	Powers
Pos.& Neg.	Div. (abr.)	Radicals
Add.& Sub.	Factoring	Graphs
Signs of Ag.	H. C. F.	Quad. Equa.
Lin. Equa.	L. C. M.	Formula
Mult. (gen)	Fractions	Square Root
Mult. (abr)	Sim. Equa.	

The list of texts analyzed are -

Ford & Ammerman, First Course in Algebra
Reitz, Crathorne & Taylor, School Algebra
Durrell & Arnold, First Book in Algebra
Hawkes, Luby & Tonton, New First Course in Algebra
Milne & Downey, First Year Algebra
Rushmer & Dence, High School Algebra
Edgerton & Carpenter, First Course in Algebra
Nyberg, First Course in Algebra
Williams & Kempthorne, First Course in Algebra
Wells & Hart, Modern First Year Algebra.

Comment

The amount of emphasis placed on the different topics seems to depend on the textbook writer's opinion. Therefore the results given in the five tables cannot be easily summarized. The emphasis is decidedly on manipulative skills.

Diagnostic Tests

13

Diagnostic Tests in the Teaching of Plane Geometry.
Carrie B. Ramsey. A.M. Colorado State Teachers' College. 1929

Construction of diagnostic tests, and the administration of them is the basis of this study in which 177 students of plane geometry in the Greeley High School and 32 students in college were used.

Conclusions.

I. Correlation between geometry scores and I.Q.'s was .52

II. Between teachers' marks and I.Q.'s was .43. Between test scores and teachers' marks was .47.

III. Students' errors resulted from

Confusion of terms
Failure to visualize figures in different positions
Failure to think of figures other than the conventional type
Mistakes in computation
Inability to use formulas
Basing conclusions on appearances
Using specific instead of general cases.

IV. New type tests are considered better because they -

take less time to administer
measure a great many abilities
reveal difficulties which can be corrected by remedial teaching before it is too late
are liked by students
are more objective
are more reliable.

Comment.

177 pupils are not enough to furnish conclusive evidence as to the value of these tests, although most teachers probably agree that new type tests are better for the reasons given.

Pupil Attitude

14

A Study of Some of the Factors Determining the Pupils' Attitude Toward Mathematics in the Secondary Schools. Howard Emerson Timbers. A.M. University of Washington. 1927 60 pages.

The Study consists of two parts; the first gives a survey of four studies made in this field from 1918 to 1922, the second part is a study of the reactions of Seattle High School students. Part II is based on returns of a questionaire sent out to 2000 high school graduates in June 1927. There were 160 replies. The questionaire consisted of 16 parts asking pupils to underline subjects taken, best liked, grades made, and to state reasons for liking or disliking mathematics.

Conclusions.

The Seattle High School students liked mathematics. For 20% it was a favorite subject. A smaller group hate mathematics.

The results seem to agree with those of the former studies reviewed.

1. Mathematics is found usually to rank first as a favorite subject.
2. There is no sex difference.
3. The reasons why students like mathematics are;
   a. It will be of practical value to them.
   b. They are interested in mathematics.
   c. They like the teacher.

Comment.

It would be interesting and valuable to learn the real feeling of mathematics students toward this subject. It seems, however, impossible to draw the conclusion that the Seattle High School pupils like mathematics from 160 replies to 2000 questionaires sent out. One must remember it was a favorite subject of 20% of 160, not of 2000.

If we consider these replies valuable, it is interesting to note that they agreed with the studied reviewed by the author.

CHAPTER II

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS YEARBOOKS 1926-1931

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

FIRST YEARBOOK

1926

A General Survey of Progress in the Last Twenty-five Years.

15

A General Survey of the Progress of Mathematics in our High Schools in the Last Twenty-five Years.

By David Eugene Smith. pp.1-31

I. The situation at the beginning of the century.

A. At the beginning of the century each college set its own requirements disregardless of other colleges, or the secondary schools, or the needs of the public.

B. In 1902 the American Mathematical Society defined what should be college entrance requirements. Before this time other opinions had been local.

C. In 1900 the College Entrance Examination Board was organized. Its purpose was to unify the examinations.

D. The mathematical material taught in high schools was of the traditional type.

II. Improvements.

A. The International Commission on the Teaching of Mathematics was established in 1908. The work of this Commission showed that the United States was decidedly behind other countries in regard to subject matter.

B. In 1916 the National Committee on Mathematical Requirements was established. This Committee published a report, "The Reorganization of Mathematics in Secondary Education". Colleges and secondary schools cooperated in preparing this report. Its influence toward improvement of instruction is keenly felt.

C. The College Entrance Examination Board revised its requirements in 1922-1923. It can no longer be said that the colleges make unreasonable demands of the high schools.

D. The rise of the Junior High Schools is one of the outstanding events of the first quarter of the century. This allows for fusion of courses.

E. Universities established schools of education for the purpose of experiment and study.

F. Text book writers have endeavored to make their books fit the needs of the pupils.

G. The progress of algebra is in accordance with the "Reorganization of Mathematics in Secondary Education".

H. There is a tendency toward fusion of plane and solid geometry.

I. The advance in testing abilities and achievements has been great.

J. The advance in the teaching of mathematics has been more marked than of any other subject of elementary or secondary education.

16

On the Foundations of Mathematics

By Eliakim Hastings Moore pp. 32-57.

This is the presidential address delivered before the American Mathematical Society in 1902, the influence of which has been so keenly felt.

17

Suggestions for the Solution of an Important Problem That Has Arisen in the Last Quarter of a Century.

By Raleigh Schorling pp. 58-105.

I. Low mastery of subject matter in schools.

A. The author reports on the results of an Inventory Test in arithmetic. The test consisted of 125 elements chosen on basis of (1)Appearance in courses of study, and (2)the judgments of 29 writers of arithmetic text books. It was an untimed test in which 3260 seventh grade pupils were tested. The summary table shows 40% to 50% mastery.

B. Surveys and tests by Thorndike, Courtis, Woody show very mastery of algebra by ninth year pupils.

II. To improve the situation.

A. "We need to formulate a basic philosophy for the guidance of our work."

B. "We need to formulate in a specific way the outcomes we desire and the degree of mastery for which we strive."

C. "In selecting our specific objectives we need to place emphasis on those elements of the curriculum, the value of which can be actually proved by one or more objective studies in the use of mathematics."

D. "We need to employ certain well accepted principles from the psychology of drill."

E. "We need to construct our teaching materials under precisely controlled and tested conditions."

Text books should not be published until they have been experimentally used.

F. " We need a basic introductory course in general mathematics in grades seven and eight and for certain groups in the ninth grade."

III. Discussion based on author's experience of each of the six preceding items.

18

Improvement of Tests in Mathematics

By W.D.Reeve pp. 106-140.

I. Means of improvement.

A. The traditional "Essay Type" tests are necessarily confined to a very snall part of the material to be tested.

B. For improving instruction diagnostic tests are of great value.

C. The class room teachers are best qualified to make tests that have both measuring and diagnostic value.

D. Standardized tests have tended to use traditional material instead of including more valuable material now set by modern objectives.

E. In constructing standardized tests the class room teachers and the makers of tests must cooperate.

F. We must first determine the purppses of teaching mathematics, decide on the material best suited to accomplish our aims, and then decide on the kinds of tests that will give the best check on our results.

II. Guiding principles underlying good tests.

A. Aim for increase of ability to master subject matter.

B. Emphasis on the fundamental parts of the subject matter.

C. Scoring should be objective.

D. Every test should be reliable.

E. Every test should be self administering, "can be easily given and easily scored."

F. Every test should set some standard of achievement for a pupil within his own group or against his own record.

III. Superiority of New Type Tests over Standardized Tests.

A. "They will enable us to cover a greater range of skills and abilities in the same length of time."

B. "They will be more easily made into a teaching device which can be used for both instructional and drill purposes."

19

The Development of Mathematics in the Junior High School

By William Betz pp. 141-165.

I. A history of the Junior High School movement.

II. Necessity of a definite curriculum in Junior High School.

A. The author feels that more experiments should be completed before such an outline can be made valuable.

20.

Mathematics and the Public

By H.E.Slaught pp. 186-193.

I. In war.

A. Mathematics rendered indispensable services during the world war.

II. In science.

A. Charles P.Steinmetz used higher phases of mathematics in his work in electrical science.

B. The Rockefeller Foundation has recently extended its range of fellowships to include mathematics as fundamental to physics and chemistry.

C. Biologists and biometricians use a wide range of mathematical subjects in research work.

D. Einstein has interested the public in his theory

of relativity.

III. In industry.

A. The great industries and large business corporations are using mathematical research in obtaining higher degrees of efficiency.

21.

Recreational Values Achieved Through Mathematics Clubs in Secondary Schools.

By Marie Gugle and others.

I. Purpose of clubs.

A. These clubs increase interest and appreciation of mathematics and also give opportunity for social life.

II. Suggested program.

A. A biographical sketch of some great mathematician.

B. A mathematical game, trick, or fallacy, or unique solution.

C. An account of some scientific discovery or invention related to mathematics.

D. The bibliography gives reference books and magazine articles from which material may be selected.

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

SECOND YEAR BOOK

1927

Curriculum Problems in Teaching Mathematics.

Junior High School Mathematics.

22.

Reorganized Mathematics in the Junior High School, Grades Seven and Eight.

By Harry C.Barber pp.125-142.

I. Essentials for achievement of aims.

A. Teachers must set up situations which seem real to the child.

B. The explanation rather than the rule must be repeated. Correct responses from the child must come because the child understands.

C. Intelligent understanding is more important than drill.

D. The work of each year should begin with new and interesting material to show the value of mathematics.

E. Pupils in vocational courses should not be given work in an extremely narrow field, but a broad training should be given.

F. Important processes, methods, and items of information should be used over and over again in the items studied.

II. Aims.

A. Accuracy.

1. Drill is not the only means of obtaining accuracy. Reclarification rather than mere repetition is the rule where understanding is desired.

2. Number judgment, not dependence on the mechanical rule must be persistently worked for.

B. Metric system. The author believes that pupils should be familiar with centimeters, grams, and and liters. His reason, however, is not apparent.

C. Ratio.

1. A ratio is a number.

2. A ratio tells "how many times."

D. Algebraic analysis. Manipulative algebra is not enough.

E. Geometric facts. Space concepts, concreteness, and the introduction to the elements of a new field of thinking.

F. Approximate numbers.

1. Numbers that come from measuring are approximate.

2. In computing with approximate numbers the results can be no more accurate than the data.

3. Computations based on three-place tables must not be relied on.

G. Statistics. Pupils must understand the convenience of tabulation.

H. Percent.

III. Ninth Grade Algebra.

A. Every algebra teacher should be familiar with the field of natural sciences.

B. Outline of study for ninth year.

C. We should go only as far as the child understands, not as far as we can show him how to manipulate.

23.

Effect of Varying the Amounts of Arithmetic, Geometry and Algebra Studied in the Junior High School.

By Ralph Beatly. pp. 143-153.

I. Report of an experiment in which the first half of Grade VII was devoted to intuitive geometry, and the first half of Grade VIII to algebra. The last half of each year was devoted to arithmetic.

The results were measured by the Peet-Dearborn Arithmetic Test. This was done in three successive classes at the end of Grade VIII. The control group was the class of the year previous to these experiments.

II. Conclusions.

A. As arithmetic is taught at present two years in the Junior High School are no better than one.

B. Algebra taught early in the eighth year clarifies later work in arithmetic.

C. Teachers in the Junior High School cannot be expected to get a wide enough background in too many different subjects, but without this background a just presentation of the subject cannot be made.

III. Further conclusions drawn from author's experience rather than from data of experiment.

A. We need at least an aggregate of one year of arithmetic in grades seven and eight.

B. To get greater returns from arithmetic in these grades we need new methods of drill and greater interest rather than more time.

C. A whole year of algebra in Grade VII is too much.

D. It is possible to teach some numerical trigonometry in Grade VIII if desired. It is possible to teach the solution of the oblique triangle in Grade IX using logarithms.

E. For effective teaching of mathematics in the Junior High School we need a certain degree of departmentalization with adequate supervision.

24

Problems in the Field of Junior High School Mathematics.
By C.L.Thiele pp. 154-172.

I. A survey of four texts of Junior High School Mathematics of wide usage.

A. The problem is "what to teach."

B. The answer should be in accordance with aims set down by the National Committee requirements.

1. To continue training in the fundamental operations.

2. To train pupils for vocational life.

3. To train pupils in their social-civic duties for worthy home membership and citizenship.

II. Bread and butter training for one pupil will be appreciation for another.

III. Outline of mathematics program for seventh, eighth, and ninth grades under headings of practical use and appreciation.

A. Consumption. Home budgeting.

B. Production.

C. Group organization, social, political, and economic.

D. Expenditures for education.

judiciary functions
charity
good health
recreation
public lighting
fire and police protection
good roads
trade

transportation

communication

Such education aims to make the masses more intelligent members of organized society.

IV. Need of protective mathematics for pupils since the public is still easily duped.

V. Less emphasis placed by author on intuitive geometry in these grades.

VI. Method of teaching such a program.

A. Lesson plans submitted must be based on a real life situation.

B. Problem of individual differences is allowed for.

C. There is a need for reteaching work of first six grades where there was only half mastery.

25.

Objectives In the Teaching of Junior High School Mathematics By David Eugene Smith and William David Reeve pp.173-227.

I. Specific objectives.

A. To give pupils some idea of the general nature and uses of business arithmetic, intuitive geometry, practical algebra, the simplest part of trigonometry, and a knowledge of the meaning of a demonstration.

II. General objectives.

A. Habits.

B. Modes of thought.

C. Appreciation of mathematics as a useful art and as a science.

D. Appreciation of history of mathematics.

E. Attitude of mind.

F. Ideals to be cultivated.

II. Concepts. List of concepts in each branch of elementary mathematics.

III. Abilities in each field of mathematics.

A. Ability in algebra includes that of solving complex fractions

$$\frac{a+\frac{b}{c}}{a+\frac{b}{c}}$$

B. Ability in demonstration is extended to include proofs of similar triangles theorems and proof of Pythagorean Theorem.

IV. Summary of objectives.

A. The survey of the field of mathematics should show a wide surface rather than a narrow depth.

B. Interest should be extended.

C. At the end of the Junior High School the pupil should know whether he is gifted in mathematics enough to pursue his studies further.

26.

Mathematics In the Senior High School

By David Eugene Smith pp. 231-241.

I. Aims.

A. The Senior High School should aim to meet the

needs of the pupil who likes mathematics and who wishes to continue its study.

B. The aim of Grade X is that of logical reasoning. In this grade geometry is the central feature, but algebra and trigonometry are brought in when advantageous. A commercial algebra course is suggested.

C. Grade XI. More trigonometry. Intermediate algebra. There has been no exploration in this field.

D. Grade XII. Only that work that is best for general knowledge. Analytic geometry, calculus.

27

The Mathematics Curriculum in the Senior High School

By Eugene R.Smith pp. 242-255.

This is a discussion of the Senior High School mathematics courses as based on the Recommendations of the National Committee. The suggestion is made that secondary algebra be placed in Grade X rather than in Grade XI to give greater continuity that makes for greater mastery.

28

A Reorganized Course of Study for Senior High School Mathematics

By Gertrude E.Allen pp. 256-290.

This is an outline of courses in the Oakland, California high schools in accordance with the National Committee requirements.

Provision is made for individual differences by setting a minimum course of study for the less gifted.

Their algebraic work included applications from permutations, probability and chance, theory of limits, mathematical induction, imaginaries, cube root, statistics, differentiation, and integration of rational integral functions.

There are also one-semester courses in trigonometry combined with solid geometry and introduction to elementary calculus, analytic geometry, and descriptive geometry.

There is a one-semester course in analytic geometry and social and economic arithmetic.

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

THE THIRD YEARBOOK

1928

Selected Topics in the Teaching of Mathematics

29

The Fallacy of Treating School Subjects as Tool Subjects

By Charles H.Judd

I. The author strongly objects to the classification of arithmetic as a tool subject. Instead he considers it a mode of thinking.

II. The author believes very firmly that mathematics instruction should not be limited to the simple uses of arithmetic as compiled by a survey showing uses parents make of arithmetic in the work-a-day world.

30

Mathematics in the Training for Citizenship

By David Eugene Smith

I. Purpose. To consider the mathematics that the working man needs.

II. Reasons for a knowledge of mathematics by everyone.

A. World uses mathematics extensively.

B. Mathematics should enter into the making of a citizen because it has high value as a mental discipline.

C. The history of mathematics is the history of the race.

D. The proper study of mathematics gives one a deeper religious sense than could be developed without it.

E. It makes man conscious of his position in the universe anout him.

F. A child should be led to the great mathematical truths because they arouse his soul to a contemplation of the truths that endure.

III. Mathematics is poorly taught, but no more poorly than pedagogy. Failures also occur in the teaching of religion, art, as well as of everything.

IV. We should cease marking time in Grades VII and VIII, and introduce intuitive geometry in these grades. We must require that the pupil take a brief introductory course in mathematics as suggested for the ninth school year.

V. All that has been said so loudly against geometry may be said with equal energy against music, physics, history, graphic arts, in fact all subjects in the school curriculum.

VI. A wider outlook is what is needed, since we too often use our time and energies puttering about as to the statistical results of teaching by one method or another, yet research has very little to offer us.

31

Mathematics as an Interpreter of Life

By W.S.Schlaugh pp. 24-34

This discussion is based on the teaching experience of the author.

I. Importance of the subject in curriculum measured by its contribution to

A. Direct self-preservation.

B. Indirect self-preservation or the earning of a living.

C. Social efficiency and citizenship.

D. The pleasure of the individual.

E. A general understanding of and an insight into economic, social, and cosmic forces and problems the mastery of which is necessary for the continuing of human progress.

II. Some outstanding topics.

A. The interest formula.

1. Value of formula and implication of results.

2. Meaning of symbols more inportant than knowing how to use symbols.

B. Binomial theorem.

1. Bonds, interest, investments.

2. In finding roots.

C. Theory of Exponents.

1. Theory of logarithms.

2. Computation by logarithms.

3. Slide rule as application of logarithms, used in problems dealing with business, accounting, and the economic world.

D. Use of graphs.

1. Approach problem through statistical graphs and graphs for calculation.

Problems connected with building, railroad time tables, points of maximum net return, etc. more easily solved by graphic methods.

E. Use of geometry in Calculation Charts.

1. Paymasters' wage alignment charts.

2. Addition, subtraction chart.

F. Use of geometry in geography and astronomy to find latitude, etc..

G. Geometry of the sextant.

H. Problems of finance and investments.

1. Installment payments with interest.

2. Amortization and borrowing.

3. Sinking fund calculations.

4. Bond valuation.

5. Trends and cycles in business.

I. Value of mathematics in higher sense.

1. "Conception of an ordered, lawful universe which all phenomena yield to quantitative investigation, a universe in which the reign

of law is absolute and all phenomena in space-time world fall into order in a cause and effect nexus."

2. "Mathematics transforms, sobers, exalts, as well as glorifies our view of reality."

32

The Reality of Mathematical Processes

By E.R.Hedrick pp. 35-41.

I. Real values of mathematics other than practical applications.

A. Precise statements, precise reading.

B. Generalization.

C. Necessary and efficient conditions.

D. Functional thinking.

E. Effect upon a completed result of error in measurement.

F. Logical proof.

G. Idealization.

II. Transfer of training seems to be more possible with respect to processes than with respect to facts and skills.

33

Developing Functional Thinking in Secondary School Mathematics

By E.R.Breslich pp. 42-56.

I. No opportunity should be missed to call pupils' attention to relationship if functional thinking is to be

cultivated. The analysis of two algebra texts and of two geometry texts show that functional thinking is practically disregarded by text book writers.

II. Functional thinking cannot be taught by means of isolated chapters.

34.

Dynamic Symmetry

Marie Gugle pp 57-64

I. Relationships in geometric basis of classic art of the fifth and sixth centuries are

A. Static symmetry of classic art.
Dimensions of its designs are commensurable.

B. Dynamic symmetry.

1. Dimensions are incommensurable. Most common ratios are $1:\sqrt{2}$; $1:\sqrt{3}$; and $1:\sqrt{5}$.

2. Dynamic symmetry is the key to the maple leaf. Symmetry of dragon-fly is root-three rectangle. Iris also root-three rectangle.

3. Every modern art school teaches this symmetry.

35.

Introductory Calculus as a High School Subject.

By M.A.Nordgaard pp 65-101.

I.. Calculus should have an earlier place in the curriculum. History shows that arithmetic, geometry, trigonometry, algebra, and analytics have moved down as

the general level of education has been raised.

II. Justification of a course in calculus in the high school.

A. Since functional thinking is now being stressed as the core of mathematics teaching, instruction should be rounded off with the notions of calculus in order that the full significance of function may be appreciated.

B. It is fast becoming necessary for an educated person to know some calculus.

1. Physics, mechanics, engineering depend on the calculus.

2. The results of the latest researches in radio-activity, medicine, and actuarial science are stated in calculus form

36

Selected Topics in Calculus for the High School

By John A.Swenson pp. 102-134

I. Preliminary subject matter.

A. The function concept.

B. Variation.

C. Locus.

D. Pupils must be able to tell

1. On what other variables a certain variable depends.

2. When two variables vary in the same sense or in the opposite sense.

3. How the various parts of a geometric figure vary together.

II. Topics in calculus which are suitable for high school, Grade XII, with illustrations as to how they may be taught.

A. Important terms and symbols.

1. Symbol $\triangle$ used to express rate of change.

B. Use of the change ratio in graphing.

C. Slope of a line.

D. The distance formula.

E. Increasing and decreasing functions.

F. The cubic function.

G. Areas by summation.

H. Volumes by summation.

I. What $\frac{\triangle y}{\triangle x}$ represents in a quadratic function.

J. The mechanical way of finding $\frac{dy}{dx}$.

K. The meaning of $\frac{dy}{dx}$

L. Maximum and minimum.

M. The inverse of differentiation.

N. Area by integration.

O. The sign of integration.

P. Integration is a summation.

37

Teaching Thrift Through the School Savings Bank

By Clifford B.Upton pp. 135-140

Schools must establish the habit of thrift by using the laws of learning.

38

Measurement and Computation

By George W.Finley pp. 141-148.

I. All numbers which result from measurement are approximate. Their accuracy depends on

A. Our measuring instruments.

B. The degree of accuracy demanded by the purpose of the measurement. A carpenter doing inside work need not be as accurate in measuring as the machinist who is making fitted parts for a typewriter.

II. In order to deal with approximate numbers it is necessary for pupil to know

A. Numbers derived from measurement are approximate.

B. The meaning of significant figures.

C. How to perform the four fundamental operations with approximate numbers.

D. Which numbers are approximate and which are exact.

E. To what degree of accuracy measurements should be obtained.

1. The question of the number of decimal places to be retained versus the number of figures to be retained should be considered.

39

The Teaching of Direct Measurement in the Junior High School
By William Betz pp. 149-194.

I. A history of the struggle of man to obtain a standard of length in terms of a natural unit reproducible at any time, the meter, in terms of wave-lengths of light.

II. Methods of classroom procedure to teach the fundamental skill of accurate measurement.

A. The study of the instrument.

1. Acquaintance with the ruler and its units.

B. Practice in crude and refined measurement.

C. Bar graphs and line graphs.

D. Method of rounding off.

E. Measurement projects.

40

The Use of Measuring Instruments in Teaching Mathematics
By C.N.Shuster pp. 195-222.

I. Importance of measuring instruments.

A. To make work real and practical instruments are as necessary as they are in physics, chemistry, or biology.

II. Description of both simple and complicated instruments with illustrations of their use. Angle mirror, hypsometer, clinometer, heliotrope, transit, compass, the plane table, slide rule.

41

A Mathematical Atmosphere

By Olive A. Kee pp. 268-276.

I. Mathematics Posters.

A. Illustrating the uses of mathematics in society.

B. Showing geometric forms in nature and art.

C. Warning pupil to heed some rule in connection with his work.

II. Graphs.

A. Constructed by pupils.

B. Cut from papers. Care should be taken to have pupil understand them.

III. Newspaper clippings.

A. Should be meaningful.

B. Should be commented on before being posted.

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

THE FOURTH YEARBOOK

1929

Significant Changes and Trends in the Teaching of Mathematics Throughout the World Since 1910

42

Austria

By Dr. Konrad Falk pp. 4-11

I. A unified type of instruction in the Volkschule.

A. Education compulsory through the age of 14 years.

1. Mittelschule, ages 10-14.

2. Oberschule, ages 14-18.

a. Four types of Oberschule, Classical, modern languages, mathematics, science, or the Deutsche Oberschule.

II. Instruction in geometry begins in the first school year. It is called the cultivation of space perceptions.

III. Teachers warned against the too early introduction of the abstract, against mechanical drill on rules, and against the use of definitions in set forms. Concrete problems are given as long as possible. Children measure, cut out, fold, construct, and draw, then develop rules, and find terms to express them.

IV. Special stress placed on the development of special concepts, and on functional thinking.

V. Collections of problems occupy the chief place in the textbook. Formal textbook material is in the appendix.

VI. Teacher training.

Teacher' Institute gives courses in review, insight into the study of the properties of number equations, higher equations, methods of approximation, functions and series, the calculus, and teaching problems.

43.

Czechoslovakia

By Dr. Guido Vetter pp. 12-23

The Mathematics Curricula of the Gymnasium.

I. First Form.

A. Geometry.

B. Work on completion of axes and planes.

II. Forms two to six.

A. Work with four place logarithmic tables.

B. Compound interest calculations.

C. Fundamentals of plane trigonometry.

III. Seventh Form.

A. Combinations.

B. Binomial theorem for positive integral powers.

C. Fundamentals of probability.

D. Insurance mathematics.

E. More plane trigonometry.

IV. Eighth Form of the Natural Science division.

A. Binomial equations.

B. Geometrical representations of complex numbers.

C. De Moivre's formula.

D. The fundamentals of infinitessimal calculus are suggested.

44 47

England

By G. St.L.Carson pp. 24-33.

I. Admission to the state-aided secondary schools is by examination. Classes are therefore more homogeneous.

II. Curricula.

A. The mimimum course in arithmetic now includes logarithms.

B. Algebra includes graphs, but goes no further than quadratics and progressions.

C. Geometry includes Euclid's first six books.

D. Trigonometry includes only the sinplest formulas and indentities.

III. Provision is made for more intensive work for pupils of outstanding mathematical ability.

IV. The work that can and should be given to pupils in a course of three or four years ending at fifteen years of age and followed by employment is a new and urgent problem.

45

France

By Monsieur A. Chatelet pp34-41.

I. In the Elementary Schools (6-11 years of age) arithmetic is the main subject in mathematics. Geometry is a study parallel to drawing.

II. Technical education (11-15 years of age)

A. The instruction is given with view to a trade, and by means of that trade.

B. Effort is made to connect instruction in descriptive geometry with the drawings of machines, geometry with workshop problems, algebra with commercial problems.

III. Higher Elementary Education (ages 11-15 years) includes courses in algebra, plane and solid geometry, and arithmetic.

IV. The Secondary School Course of seven years prepares for the University and includes trigonometry and descriptive geometry.

46

Germany

By Dr. W.Lietzmann pp. 42-52.

I. Grundschule (four years) is compulsory. Following this pupils may enter Volksschule (4 years), Mittelschule (6 years), Höhere Schule (9 years)

II. Contents of mathematics for these schools.

A. Grundschule. Four fundamental operations.

B. Volkschule. Fractions and applications in practical problems. The most important concepts of plane and solid geometry. Measure of areas and volumes. Drawing ground plans and elevations.

C. Mittelschule. Plan of mathematics is the same as that of first six grades of higher schools.

D. Higher Schools. Based on the 3-3-3 plan.

1. First three years. uses of the four fundamental operations with whole numbers and fractions.

2. Second three years. Raising to powers. Roots. Logarithms. These are developed in connection with numerical work in the study of equations of the first and second degree.

Functional dependence and graphic representation is important.

Plane and solid geometry also studied.

3. Last three years. Complex numbers and De Moivre's Theorem. Arithmetic geometric progressions. Reckoning of interest, annuities, insurance. Spherical trigonometry with applications to geodesy and astronomy. Analytic geometry. Map making.

47

Holland

By Dr. J.E.Schrek pp53-62.

I. In elementary education (ages 5-11) there have been no important changes since 1910.

II. In secondary education (ages 12-18) important changes were made by royal decree in 1919.

III. Curricula.

A. First four forms have algebra and geometry.

B. The fifth and sixth forms have the more difficult algebra and geometry, arithmetic and geometric series, logarithms, plane trigonometry, analytic geometry including conic sections, elements of the calculus.

IV. Practical applications of elementary mathematics have been neglected. The slide rule is unknown in teaching. Want of improvement is keenly felt.

48

Hungary

By Professor Charles Goldziher pp. 63-70.

I. High Schools (ages 10-18)

A. Graphic work introduced in arithmetic.

B. Function concept introduced in beginning algebra.

C. Real measuring and numerical computations.

D. Analytic geometry and calculus introduced in seventh class.

E. Uses of mathematics in physics, engineering, and economics.

F. Fusion of algebra and geometry.(Geometry appears as a separate subject only in the Realschule.)

II. The Realschule has a course in descriptive geometry in grades 5-8.

49

Italy

By Professor Federigo Enrigues pp. 71-76

I. Schools are expected to train for attitudes rather than to furnish facts.

II. High schools prepare for the university, and pupils must pass state examinations.

III. The mathematics course is in two parts.

A. Algebra, trigonometry, exponential equations, logarithms, periodic functions and applications.

B. Plane and solid geometry. Rigorous deductive proof.

50

Japan

By Professor Yayotaro Abe pp77-93.

I. Elementary schools, six years in length, are compulsory.

II. Higher elementary school of two years, or a secondary school of five years is optional.

III. Curricula.

A. Middle school, a five year secondary school.

1. First year. Arithmetic.

2. Second year. Algebra,

3. Third year. Algebra and geometry.

4. Fourth year. Algebra and geometry.

5. Fifth year. Algebra, geometry, and trigonometry

49.

50

B. Higher middle school of two years. This prepares for the university.

1. Solid geometry.
2. Analytical geometry, plane and solid.
3. Algebra.
4. Calculus.
5. Dynamics.

51.

Russia.

By Professor D.Sintzof pp. 94-111.

I. "Mathematics is a discipline, practical and necessary. The utilitarian value in mathematics and the necessity for training in mathematics constitute its justification."

II. Secondary schools. (Grades 5-9)

A. Unified mathematics is taught taking the easiest of all in the lower grades.

III. In the ninth year.

A. Arithmetic and geometric progressions.
B. Variation.
C. Limits.
D. Irrational numbers.
E. Regular polygons.
F. Cavalieri's Theorem applied to volumes of prisms and pyramids.
G. Volume of curved surfaces.

H. Trigonometry.

I. Equations of higher degree.

J. Exponential and logarithmic equations.

52

Scandanavia

By Professor Paul Heegaard pp. 112-119

I. In the technical school no significant changes made since 1910.

II. Act of 1910 sets the following objectives for all kinds of mathematics.

A. Continuation of arithmetic.

B. Algebra and plane geometry.

C. Trigonometry.

D. Problems in construction and calculation.

III. In the Reallinjen.

A. Advanced algebra and trigonometry.

B. Solid geometry.

C. Conic sections.

D. Descriptive geometry.

E. Elementary calculus and its applications may be elected.

53

Switzerland

By Professor S.Gagnehin pp. 120-130

I. The syllabus of 1925 shows a reduction in required subject matter.

II. The secondary school course which leads to a diploma in the mathematics course includes

A. Theory of numbers.

B. Advanced algebra.

C. Geometry.

D. Trigonometry.

E. Analytic geometry.

54

The United States

By Professor William David Reeve pp. 131-186.

I. A history of the reconstruction movement and the reorganization of mathematics in accordance with new aims, content, methods, and testing program.

A. A problem peculiar to the United States is the increase in school population. In 1918 the number graduated from high schools was eight times the number in 1890.

B. Algebra has been simplified.

C. Functional thinking is being emphasized.

D. The possibility of a course in calculus in high school is being considered.

II. An outline of our course of study as given in The Reorganization of Mathematics in Secondary Education.

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

THE FIFTH YEARBOOK

1930

The Teaching of Geometry

55

The Teaching of Geometry

By William D. Reeve pp. 1-28.

A summary of the best practices in present day teaching with respect to approach, content, and classroom methods.

56

What Shall We Teach in Geometry

By W.R.Longley pp. 29-38.

I. Reasons why geometry should be taught.

A. Logical exposition.

B. Geometric facts and relations.

C. Mensuration formulas and methods.

D. Cultivation of space perception.

II. A discussion of a possible course in solid geometry which would be less of the traditional sort, and still prepare for the College Board Examinations.

A. At present the best course is to spend as little time as possible on propositions of solid geometry and more time on original exercises.

57

A Unit of Demonstrative Geometry for the Ninth Year

By Joseph Seidlin pp. 54-63.

I. Purpose of the unit is to show the pupil what demonstration means.

II. A list of proofs of five propositions is given.

58

Teaching Plane and Solid Geometry Simultaneously

By May L.Wilt pp. 64-66.

I. A discussion of the possibility of the simultaneous presentation of both plane and solid geometry in case of the following topics:

A. Definitions.

B. Congruency of triangles.

C. Parallel lines.

D. Volumes and areas.

E. Circles.

F. Loci.

II. This enlargement of ideas tends to make pupils more "space minded" instead of "plane minded".

59

Demonstrative Geometry in the Seventh and Eighth Years

By Vera Sanford pp. 39-43.

I. In the author's opinion the seventh or eighth grade is not too early to introduce idea of a proof by reasoning.

5-7

5-8

A. Opportunities may come by chance or be introduced by suggestion.

B. This makes for unity in the geometry of the Junior High School.

C. It bridges the gap between informal and demonstrative geometry.

60

A Unit of Demonstrative Geometry for the Ninth Year
By Joseph B.Orleans pp. 44-53.

I. In the author's opinion demonstrative geometry can best be introduced in the ninth year if algebra has been begun in the eighth year, and if intuitive geometry is introduced in the seventh year.

II. This unit(six weeks) consists of a series of exercises which are easily demonstrated.

A. Special emphasis laid upon certain important problems.

B. Content. Notions of congruence, symmetry, parallelism, and similarity.

61

An Experiment in Redistribution of Material for High School
By Gertrude E.Allen pp.67-85.

I. An outline of the course of study in elementary geometry(tenth year)

II. Statements of four different methods for unifying plane and solid geometry.

III. Statement of a plan for combining some trigonometry and arithmetic with work of plane geometry. The theories and practices used can be defended by statistics furnished from the Oakland Department of Research and Guidance.

62

A New Approach to Elementary Geometry

By George D.Birkhoff and Ralph Beatly pp. 86-95.

I. An approach based on the number system, using the elementary ideas of analytic geometry. The theorems of geometry can be proved and formulated in terms of concepts of analytic geometry.

II. A modification of Euclid in accordance with psychological and mathematical ideas which were not available to Euclid.

63

Graphic Methods of Teaching Congruence in Geometry

By John A.Swenson pp. 96-101.

Four diagrams showing that geometry as well as other branches of mathematics is fundamentally a study of variations. This graphical presentation would tend to fuse algebra and geometry.

64

The Use of Indirect Proof in Geometry and in Life

By Clifford Brewster Upton pp. 102-133.

I. The writer is convinced that geometry instruction should include a unit on indirect proof. This type of proof will carry over into life situations.

II. The writer feels that the reason why indirect proof has not met with the favor of teachers and pupils is that with our present sequence of propositions the indirect method is introduced too early, and that the unit of instruction does not give practice enough.

III. Consideration of the main objectives of teaching geometry, the teaching of proof and of deductive reasoning.

65

The Analytic Method in the Teaching of Geometry

By W.S.Schlaugh pp. 134-144.

I. The author feels that the analytic method is the heart of geometric reasoning.

II. In order to obtain sufficient comprehension and mastery a full year must be devoted to the subject.

III. If plane and solid geometry are combined into a one year course, the student's view of both subjects will be indequate and the chief value of the logical training sacrificed.

63

66

Symmetry

By John W. Young pp. 145-148

I. The use of the idea of symmetry in geometry instruction.

A . Angle bisection.

B. Construction of perpendicular bisectors of line segments.

67

Some Desirable Characteristics in a Modern Plane Geometry Text

By L.E.Mensenkamp pp.199-206.

I. A valuable outline of points to be considered in evaluating text books in geometry.

A. No attempt is make at scoring.

B. The various characteristics do not have the same value.

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

SIXTH YEARBOOK

1931

Mathematics in Modern Life

68

The Application of Mathematics to the Social Sciences.

By Irving Fisher pp. 1-17.

I. Mathematics in economic theory.

A. Supply and demand.

B. Determination of prices.

C. Theoretical effect of taxes or tariffs on prices.

D. Making sure that a problem is determinate.

E. Illustrating geometrically or analytically the fact that price or a marginal utility is a function of many variables.

F. Helping to make clear the "dimensionality" of the magnitudes treated.

G. Helping to analyze time relationships.

II. The smoothing of Statistical Series, the fitting of formulas and curves to statistics.

A. Mortality tables.

III. Correction.

IV. Analysis of Probability.

V. Index numbers in the field of economics and statistics.

VI. Mathematical notation is required for the best expression of scientific method.

69

Mathematics in Biology

By J.Arthur Harris pp. 18-35.

I. Biology is the last of the sciences to adopt mathematical methods. As the science becomes more perfect, it tends to become more mathematical.

II. Mathematics will have an increasing influence in determining the direction of research.

70.

The Humanistic Bearings of Mathematics.

By Cassius J.Keyser pp. 36-52.

This article is really the philosophy of mathematics.

I. To deal with world of ideas the discipline of logic and mathematics is necessary.

71.

Mathematics and Religion

By David Eugene Smith pp. 53-60

This is also philosophy of mathematics.

I. Study of mathematics gives one some religious insight.

72.

The Mathematics of Investment

By William L.Hart pp. 61-85.

I. Mathematical prerequisites.

A. Arithmetic.

B. One and a half years of high school algebra.

72 con't.

II. Mathematics in investment.

A. Simple interest and simple discount.

B. Compound interest.

C. Annuities certain.

III. The author believes that a large part of the mathematics of investment could be included in a secondary school in arithmetic.

73.

Mathematics in Agriculture

By Harry Burgess Roe pp. 86-101.

I. The business of farming involves a practical knowledge of mathematics of a wide scope. It includes

A. Mensuration.

B. Proportion.

C. Rates and the compound interest idea in annuities and amortization.

D. Graphs and their interpretation.

E. Frequency distribution.

F. Application of mathematics to the elementary principles of surveying.

I I. Agricultural engineering embraces the following fields.

A. Farm Structures, covering farm buildings and other structures and related equipment, such as that for

73 con't

lighting, heating, ventilation, sanitary arrangements, and water supply.

B. Farm Mechanics, covering animal, steam, gas, and electric power, tillage machines and implements, farm home conveniences and related equipment.

C. Land Reclamation, covering land clearing, drainage, control of soil erosion, irrigation, and related structures and equipment.

III. Subject matter requirements in the secondary schools should in the opinion of the author be much the same in all courses.

A. Arithmetic.

1. A ready and correct use of the fundamentals in practical applications.

B. Algebra.

1. The author believes that algebraic notation and processes should be introduced as early as possible; at least as early as seventh grade and possibility in the sixth.

2. Elementary principles should be handled with understanding and facility.

C. Geometry.

1. Author believes that the number of fundamental propositions demonstrated in the text should

73 con't

be reduced to a minimum.

2. Original demonstrations should be encouraged.

3. No sharp line should be drawn between plane and solid geometry.

4. Practice in applications of geometry can not be overemphasized.

D. Calculus should be introduced in the high school.

74.

Mathematics in Pharmacy and in Allied Professions.
By Edward Spease pp. 102-110.

The author believes that requirements for high school graduation should include arithmetic, three semesters of algebra, plane and solid geometry.

There is little mathematics taught in the medical college. It is assumed that the student is prepared to carry out all necessary calculations.

75.

Mathematics and Statistics
By Helen M.Walker pp. 111-135.

I. Mathematical theory must be mastered.

II. The expert in statistical theory needs also a rich knowledge of the content of the field in which he would work.

75 con't

III. "The man who develops a new piece of statistical theory works as a mathematician and faces only those obligations ordinarily incumbent upon the mathematician."

IV. Differential calculus is the minimum preparation for the student who hopes to do anything at all with the theory of statistics.

V. Topics in arithmetic and algebra most needed for beginning statistics.

VI. Attitudes and habits.

A. Ability to think in terms of symbolism.

B. Correct thinking about variables.

C. Freedom from fear.

1. We should teach so that boys and girls may have the triumph of becoming mathematical adventurers and discoverers and so secure self-confidence.

D. Right attitudes toward the outcome of computation.

1. The habit of estimating in advance.

2. A conscience about accuracy.

VII. Informations and skills.

A. Mathematics through calculus.

VIII. At present many college students have inadequate background in mathematics. College courses in statistical method are increasing rapidly.

76

Mathematics in Physics

H.Emmett Brown pp. 136-164.

I. Physics now highly mathematical, therefore mathematics an invaluable tool.

A. To simplify subject and make it more intelligible.

B. To enrich the concepts of physics.

C. To make it directly applicable and useful.

D. To show the interrelations of the subject matter.

II. What physics asks of mathematics.

A. Algebra.

1. Formulas.

a. Solve numberically.

b. Solve literally.

2. Proportion and variation.

3. Other operations are satisfactorily taught at present.

4. Metric system.

5. Slide Rule.

a. Reduces amount of attention given to mathematical processes.

b. Time saver.

c. Accuracy of results.

III. Success in physics depends on interest and ability to handle mathematics.

IV. Possibility of combined course in mathematics and physics.

77.

Polygonal Forms

By George D.Birkhoff pp. 165-195.

I. A mathematical study of the empirical aesthetic formula

$$m \approx \frac{o}{c}$$

A. m aesthetic measure.

o "the order" attributed to certain aesthetic factors, such as symmetry.

c the complexity.

II. A classification of ninety forms according to aesthetic value obtained by formula.

III. Plato's triangle, that of which the double forms a third triangle which is equilateral.

IV. Golden Rectangle.

Ratio 1.618 which is nearly $\frac{8}{5}$, most beautiful rectangle.

CHAPTER IV

SCHOOL AND SOCIETY

SCHOOL REVIEW

TEACHERS' COLLEGE RECORD

JOURNAL OF EDUCATIONAL RESEARCH

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SCHOOL AND SOCIETY

1926-1931

78

History of Mathematics in the Schools
By G.A.Miller School and Society. Feb. 1926. Vol.23;p.241

In considering the question of introducing a course in the history of mathematics in the high school, the question of teacher mastery is raised. The author believes it would be difficult to find suitable teachers since the teacher should be an authority.

79

Controversial Mathematics
By G.A.Miller. School and Society. Oct. 1927. Vol.26;pp423-424.

There will probably always be controversies in mathematics. In the case of the calculus, controversies led to a more careful statement of the point in question. As long as advances are being made, there will be disputes over some of the most fundamental matters as well as over the trivial.

80

The Teaching of Junior High School Mathematics
By L.C.Karpinski. School and Society. Mar.1928. Vol.27;p.300.

The author objects to the text book on Junior High School Mathematics by D.E.Smith and W.D.Reeve on the ground

that neither of the writers teaches in the classroom. Text books cannot be written to include a desk study of material which is likely to be most valuable to the pupil if he should leave school at the end of that year.

A text book for Junior High School mathematics must be worked out in the daily contact with pupils in the classroom.

81

Modern Plane Geometry.
By J.C.Bay. School and Society. May 1928. Vol.27;p.572.

A severe criticism of the text book by Dr. John R.Clark and Dr. Arthur S.Otis on the ground that it is too formal, adds nothing to mathematical knowledge, and that the 80 challenges do not add to interest.

"A procedure called analytic proof is presented on one page, practiced on the next and then dismissed completely." The author calls this a robot procedure, and recommends the technique of investigation,discovery, and proof.

82

Algebraization of Mathematics.
By G.A.Miller School and Society. Sept. 1928. Vol.28;p.363

Newton and Leibnitz algebraized some of the earlier developments relating to the calculus and are given credit

for discovering it.

The algebraization of trigonometry led to important advances in algebra.

Algebraization tends to economize thought.

Algebraization and arithmetization have been fundamental tendencies in many of the developments of mathematics of the widest scope.

83

Useful Mathematics.
By G.A.Miller. School and Society. May 1930. Vol.31;pp.646-647.

The uses that are made of mathematics after school days depend on the intellectual atmosphere in which these days are spent. This atmosphere varies widely and depends to a great extent on earlier training. The need for mathematics increases as life becomes more complex. The amount of mathematics that will be useful in the future cannot be fully determined by what was useful in the past.

The author believes that we should improve mathematics teaching to make it more effective and not on the ground that a great deal of it is actually useless.

Study to Determine Whether Algebra Should be Taught in Ninth or Tenth Grade.
By Volney Ward. School and Society. Jan. 1931. Vol.33;pp.178-179.

This study involved 180 ninth grade and 159 tenth grade students in elementary algebra. The tenth grade students had had arithmetic in the ninth year. In each school the same teacher was found teaching both the ninth and tenth grade classes. The same text book was being used in each school. The Hotz Equation and Formula Scale and the Hotz Problem Scale were given the last week of the school year.

Equations for cumulative score on both Hotz Algebra Scales were worked for each grade, using the equation for the cumulative score for the ninth grade.

The most probable algebra score of the average ninth grade pupil is 8.5 ± 1.3. The most probable score of the same individual after he has reached the mean age of the tenth grade, found by substitution in the tenth grade equation is 8.6 ± 1.3.

FINDINGS.

Ninth year arithmetic did not function very much in the rating of the tenth year students in algebra.

Beginning algebra may be offered in the ninth or tenth grade without any serious effect on achievement.

Age(within the limits of this study) is not a substantial factor of success in first year algebra.

85

Sexagesimal Fractions.
By G.A.Miller. School and Society. Jan. 1931. Vol.33;pp.58-59.

Sexagesimal fractions seem to have come into use by the Babylonians in a desire to avoid the complex operation of adding fractions. Their unit fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ can be exactly expressed in terms of a sexagesimal fraction. The object of their so-called multiplication tables was to convert common fractions into sexagesimal fractions.

86

A New Mathematical Encyclopedia.
By G.A,Miller. School and Society. Dec. 1931. Vol.34;pp.756-758.

There is no modern mathematical dictionary or encyclopedia in English.

There have been great advances in the various branches of mathematics including the elementary field in the last fifty years. We are now forced to use a dictionary in an unfamiliar language.

SCHOOL REVIEW

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87

Diagnosing Cases of Failure in Algebra.
By Peter L.Spencer. School Review. May 1926. Vol.34;pp.372-376.

In working with a class of eight pupils who had failed in algebra a diagnostic test was constructed for each unit of work. Endeavor was made to "introduce but one difficulty at a time, and to pass from the simple to the complex through rather definite degrees of increasing difficulty."

The results show that causes of failure are easily found; that all pupils do not experience the same difficulty, and therefore that mass instruction is impossible with such a group.

88

Percentile Rank in Intelligence as a Prognosis of Success in Algebra.
By Harry E.Elder. School Review. Sept.1926. Vol.34;pp.543-546.

The purpose of this study is to determine the relation between success in algebra and the scores made on standard intelligence tests. It is based on grades of fifty ninth year algebra pupils and the percentile ranks of the same pupils in two intelligence tests.

The Mental Survey Scale No.1(Indiana University) had been given during the pupils' eighth school year. Otis Self Administering was given in the ninth grade.

The correlation between the two intelligence tests is .78 +.04.

The correlation between the scores in algebra and the average ranks obtained in the intelligence tests is .60 ± .06.

One table is given. While correlation between algebra score and percentile intelligence ranks is high, there are significant variations. Thirty-six per-cent of the pupils did not rank in the half of the algebra class indicated by their standing in the intelligence tests.

89% of the pupils with a mean percentile rank in intelligence above the first quartile of the group studied did satisfactory work.

Of the pupils with low percentile ranks on the mental tests only 31% received satisfactory marks in algebra.

Conclusion:

It seems that with the increased use of mental tests success in algebra may be predicted with a high degree of accuracy.

89

A Preliminary Study of Mathematical Difficulties.
By Wilbur Alden Coit. School Review. Sept.1928.Vol.36.pp.504-509.

An elementary algebra test given to 230 pupils, grades 9-12, show an extensive amount of failure is due to lack of

understanding of the elementary operations. Pupils in the higher grades often made the same mistakes as pupils in the lower grades. The same tests given to college students revealed that the same mistakes were again made.

A diagnosis of the causes of failures showed that pupils are unfamiliar with the seven forms subtraction may appear in. Mastery of all forms is essential.

Remedial work consisted of drill on the seven forms of subtraction, using whole numbers and common fractions. Drill was given to half the pupils in each of four schools. The other half were not drilled. After a two week period all pupils were retested. After a lapse of two months during which no drill was given, pupils were again retested.

The tables showing conparative scores between the control and experimental groups indicate that the drill was effective.

FINDINGS.

There are difficulties consisting of fundamental concepts and skills. These difficulties also occur in the higher grades.

Drill seems to be effective in mastery of fundamental concepts and skills.

The Nature of Difficulties Encountered in Reading Mathematics. By J.S.Gorges. School Review. Mar. 1929. Vol.37;pp. 217-226.

Recent studies in reading show that the high school pupils need further and differentiated training in reading.

This particular study was made in a seventh grade class in mathematics. The teacher, interested in remedial work, recorded and classified difficulties encountered by the pupils. 188 reports were analysed. These were the reports of 40 pupils. 155 reports show 218 cases of difficulty.

Difficulties in understanding and interpreting a statement.	37.2%
Inability to interpret author's illustrative material because of a lack of mathematical apperception.	21.1%
Difficulties arising from inability to analyse.	10.6%
Difficulties due to a lack of preciseness in reading.	10.6%
Difficulties due to a lack of intensity in reading.	12.8%
Difficulties arising from the manner in which exercise is stated.	7.8%

FINDINGS.

1. There are difficulties in reading due to the mathematical vocabulary.
2. To do corrective work it is necessary to know what is the type of the difficulties.
3. Systematic corrective work must be based on definite information.

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Mathematics in the Junior High School.
By John R.Clark, Lincoln School.
Teachers' College Record. Dec. 1926. Vol.28; pp. 360-373.

This course should be an overview course, bringing in arithmetic, algebra, geometry, and trigonometry. It should emphasise the power to reason and to do precise thinking rather than so much manipulation. It should also be practical in that it would prepare for actual or highly probable social needs, since it would teach the most useful skills, concepts, and attitudes. The subject should be simplified, and ahould be extensive rather than intensive.

The aims of this course should be-

Computation with 100% accuracy.

Teaching of common business practice such as budgets, simple bookkeeping, investments, insurance.

Intuitive geometry.

Attitudes and general notions that are developed.

The function concept as the central unifying principle.

The Mathematics of the Senior High School.
By W.D.Reeve, Professor of Mathematics, Teachers' College.
Teachers' College Record. Vol.28;pp.374-386.

The Senior High School mathematics course should meet the needs of the following four groups of pupils--

1. Those who intend to go on to colleges and technical schools.
2. Those who are going to specialize in commercial work that requires mathematics, especially algebra.
3. Those who expect to specialize in science.
4. Those who like mathematics enough to desire to study it further.

The senior high school should have probable foundation courses as follows, elementary algebra in the absence of a junior high school, plane and solid geometry combined into a one year course. It should have a course in intermediate algebra, which should show the notion of dependence of one quantity upon another. For those, however, who intend to proceed further in science, economics, mathematics, or other technical subjects more mathematical work is necessary. There should be a one half year course in college algebra. There should also be courses for future mathematicians. The senior high school should also offer c courses for vocations, such as a course in machine shop mathematics, one in mechanics, and one in commercial algebra. There should be general mathematics in all grades.

This study gives an outline of suggested content for general mathematics courses for grades XI and XII.

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Mathematics in the School of the Future.
By Eugene Randolph Smith, Headmaster of Beaver Country Day School, Chestnut Hill, Mass.
Teachers' College Record. Vol. 30; pp. 795-807.

Because the future will be an age of greater mechanized productions and specialization, civilization will be too complex for children to grasp. Therefore the schools must afford a higher level of education, both for ability to earn one's living and for wiser use of leisure time through the pleasure of using one's mind.

This will affect mathematics in the following ways:

1. The elementary mathematics now taught must prove its usability, and the degree of mastery necessary must be determined.
2. The general mathematics of the junior high school will be an orientation course. The other parts of mathematics must be taught with their effects on the attitudes and powers of the students always in mind.
3. There will be greater use of prognosis tests; and less mass education and more study of the individual.
4. There will be more emphasis on teacher training. Teachers must be trained to anticipate difficulties, and by

preventive rather than corrective means gradually eliminate the discouraging type of obstruction.

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Opportunities for Transfer Making in Elementary Mathematics. By Joseph Jablonower. Teachers' College Record. Vol.30;pp.808-818.

Training tends to remain specific. If transfer is to be accomplished, it must be practised. Only the genius discovers the general or universal aspect of a relationship. There is danger that the novice will make an indiscriminating transfer. In mathematics are found opportunities for making transfers as follows:

1. The method of exclusion.
2. The epithet; glorifies something not understood, as surd, transcendental, imaginary, infinity.
3. The theory of functions gives intimations of the inter-connectedness of each with all.
4. The postulates, a mathematical prejudice.

Much of the difficulty of mathematics teaching can be attributed to its extreme preoccupation with the trivialities of problem and equation solving. Recent changes in content courses indicate a movement toward types of exercises which make for transfer.

JOURNAL OF EDUCATIONAL RESEARCH

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Predicting Success in Algebra.
By Theron L.McCuen.
Journal of Educational Research. Jan.1930. Vol.21;pp. 72-74.

This study was made to determine the relative values of the following criteria in predicting success in algebra.

1. The group Intelligence Quotient as determined by the Terman Group Test.
2. Score on the revision of the Stanford Achievenent Test.
3. Score on the Arithmetic Computation Section of the Revision of the Stanford Achievement Test.
4. Score on the Arithmetic Reasoning section of the Stanford Achievement Test.
5. Score on the Number Series section of the Terman Group Test.
6. Score on the Terman Group Test.
7. Score on the arithmetic section of the Terman Group Test.

Success in algebra was measured by the Douglass Stanford Diagnostic Algebra Tests, Series A, Form I given at the end of one semester's work in elementary algebra.

There were 32 students in Group I, 24 in Group II, 28 in Group III, and 32 in Group IV. Scores on the Terman Group Test were available for all of these students. The students in each group were ranked according to their

scores on the algebra tests. The product-moment method was used in computing the coefficient of correlation for each group between the score in algebra and the score listed in paragraph one.

The correlations are not high. Those for the group Intelligence Quotient are the most consistent and give a higher average.

Group I	.54 ± 08
Group II	.47 ± 11
Group III	.34 ± 11
Group IV	.42 ± 10
Average	.42 ± 10

Conclusions:

The Group Intelligence Quotient as determined by the Terman Group Test of Mental Ability is a better basis for grouping students in algebra according to probable success than any of the other criteria used in the study.

There are too many other factors involved in predicting success in algebra. Interest and industry and attitude toward the teacher may be influential factors.

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Influence of Class Size Upon Pupil Accomplishment in High School Algebra.
By Milton B.Jensen and Dortha W. Jensen.
Jornal of Education Research. Feb. 1930. Vol.21; pp.120-137.

A summary of many studies based on opinions as obtained from questionaires shows that in the opinion of teachers and supervisors a class should not be larger than 40.

No attempt was made to control such factors as initial accomplishment, race differences resulting in language handicaps, repeaters in experimental groups, sex, and diurnal changes.

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Influence of Class Size Upon Pupil Accomplishment in High School Algebra.
By Milton B.Jensen.
Journal of Educational Research. May 1930. Vol.21;pp.337-356.

This study of class size involved 185 children, 64 girls and 121 boys in a second semester algebra group. There were no repeaters and no pupils with language deficiencies. The Terman Group Test, Intelligence Quotient was available foe all. The date of birth of each pupil was obtained. The boys and girls were separated. The girls were separated into two classes of 47 and 17. The grouping of the girls was not homogeneous. The boys were separated into two groups, those having an Intelligence Quotient above 102, and those having an Intelligence Quotient below 103. Each group was divided into large and two small classes. Success was measured by the Achievement Content Test in algebra and by the teachers' marks for two periods of time.

The conclusions drawn are that instruction in small classes is superior, and that achievement in small classes is more rapid..

Studies based on promotiom rate show very slight advantage of small and medium size classes. In all these studies there were many uncontrolled factors. A study reported in the 1929 Yearbook of Superintendence of the National Education Association shows that promotion is affected by factors other than class size.

Recent studies in elementary grades using Standardized Tests as measures show conflicting conclusions. Three studies in class size in universities show no differences in accomplishment in small and large classes.

Only one high school study is reported. This study was made by the North Central Association in 1922. It involved 100 high schools, in which there were 65 small classes of less than 20 pupils, 120 medium size classes of 30 pupils, and 55 classes of less than 50 pupils. The number of classes in each subject is not given in the data. There are no consistent general tendencies. It is concluded that class size has no important effects on school marks.

Pupils in small classes in English made higher grades than pupils in medium or large classes. Medium size classes in modern languages had the highest ratings.

Pupils in large classes in social sciences and mathematics had better marks than those in small and medium size classes.

Abilities and Disabilities of College Students in Elementary Algebra.
By H.J.Arnold.
Journal of Educational Research. April 1931. Vol.23;pp.324-329

Deficiencies in algebra is the cause of many failures in college physics, chemistry,and mathematics. Thorndyke (E.L.Thorndyke. The Psychology of Algebra. New York. The Macmillan Co. 1924) has shown that physics problems require the ability to chose the appropriate formula, to make substitutions, and to solve equations. These involve skills of clearing fractions, transposing, combining, finding square root and other fundamental skills. In chemistry the ability to form a correct proportion is of high value.

Two freshman classes of college algebra were given the Hotz first year algebra scales. 66% scored below the standard first year high school norm.

An error analysis shows the following to be most common.

1. Errors in clearing of fractions.
2. Errors due to failures to heed changes of signs in combining terms and in removal of parentheses.
3. Incorrect reduction in the case of similar fractions and fractional answers.
4. Failure to invert the divisor.
5. Inability to formulate correct equations.
6. Inability to read comparatively simple graphs.
7. Inability to indicate relationships in terms of algebraic symbols.

SUMMARY

There are 99 theses and articles reviewed in this article. Twenty-five of these are research studies, and 74 of them are discussions. Of the doctors' and masters' theses six are on curriculum and content and three on text book analyses. This shows that an effort is being made to ascertain and improve material now being taught. One of the interesting in this group is the doctor's dissertation by Clarence McCormack, Columbia University (reviewed on page three), which shows the present status of general mathematics. Teachers now prefer mathematics of a more general type through grade nine.

Two of the theses are studies to find what material increases pupil interest. The master's thesis by Louise Barbour Eddy, Motivating the Study of Geometry (reviewed on page 11) is particularly interesting because it shows how text book writers have increased the amount of space devoted to interest producing material.

The nine research studies in the educational magazines are on diagnosis and prognosis with two on class size. This again shows an effort is being made to prevent failure. All these studies are valuable.

The research studies should begin at deeper levels. Most of the present ones make use of the questionaire method, or are text book analyses. The experiments with children have not been extensive enough, and in most cases

have not been repeated. The experiments should be carried on with experimental and control groups.

Thirty of the discussions are on content. The problem is what to teach, about which there is a great variety of opinion.

Six of the discussions are on objectives. The widest difference of opinion is apparently in this field. In the crowded curriculum of the schools of the present day all material must prove its use.

The Fourth Yearbook of the National Council of Teachers of Mathematics is devoted entirely to the mathematics curriculum of other countries. Most countries evidently have a more unified type of instruction than we have, and have much more advanced work in their college preparatory schools.

In an effort to show the applications of mathematics the Sixth Yearbook, Mathematics in Modern Life, contains articles written by experts in other fields making use of mathematics. These show how extensive is the use of mathematics in the other fields.

The result of a study of this kind makes the student realize the need of extensive work in establishing the objectives of secondary school mathematics, and then determining the methods and content which will best attain these aims.

Table I, showing the classification of the 26 studies the nature of which is research.

No. of Article	Topic	Type	Source	Page
1.	Curriculum Gen. Math.	Library Study	Ph.D.Thesis	3
2.	Curriculum Gen.Math.	Minor Research	Ph.D.Thesis	8
84.	Curriculum Algebra	Minor Research	School & Soc.	102
3.	Methods Geometry	Minor Research	A.M.Thesis	11
4.	Methods Algebra	Library Study	A.M.Thesis	13
5.	Correlation	Library Study	Ph.D.Thesis	16
6.	Correlation	Library Study	A.M.Thesis	22
7.	Content Algebra	Minor Research	Ph.D.Thesis	24
8.	Content Algebra	Minor Research	Ph.D.Thesis	27
17.	Content Algebra	Library Study	National Council Yearbook	49
23.	Content Gen.Math.	Experiment	National Council Yearbook	56
9.	Objectives Geometry	Library Study	Ph.D.Thesis	29
10.	Objectives Algebra	Library Study	Ph.D.Thesis	34
99.	Diagnosis Algebra	Library Study	Journal of Ed.Research	116
87.	Diagnosis Algebra	Experiment	School Review	104
13.	Diagnosis Geometry	Testing Analysis	A.M.Thesis	44
89.	Diagnosis Algebra	Minor Research	School Review	105
90.	Diagnosis	Library Study	School Review	107

No. of Article	Topic	Type	Source	Page
88.	Prognosis Algebra	Library Study	School Review	104
95.	Prognosis Algebra	Minor Research	Journal of Ed. Research	112
9.	A Text Analysis Geometry	Analysis	A.M.Thesis	29
11.	Text Analysis Geometry	Analysis	A.M.Thesis	36
12.	Text Analysis Algebra	Analysis	A.M.Thesis	42
14.	Pupil Attitude	Library Study	A.M.Thesis	45
96	Class Size Algebra	Library Study	Journal of Ed. Research	113
97.	Class Size Algebra	Minor Research	Journal of Ed. Research	114

Table II, showing the classification of the 74 articles the nature of which is discussion.

Topic	Number of Discussions	Article Numbers in Thesis
Content and Curriculum	33	15,19,22,27,28,35,36, 38; 42-61 inclusive, 64,66,91,92,93.
Applications	14	20,30,31,34,37,68,69, 72,73,74,75,76,77.
Methods	7	33,39,55,62,63,65,94.
Objectives	6	16,24,25,26,29,32.
Motivation	3	21,40,41.
Philosophy	3	70,71,79.
History	3	78,82,85.
Critisms of Textbooks	2	80,81.
Testing	1	18.
Textbook Characteristics	1	67.
Mathematics Encyclopedia	1	86.

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